Dear Editor,

We have carefully read the comments and questions from referees A and B and your instructions regarding the figures. We have fixed the title in Figure 9 and decreased the figure file size for Figure 1 and Figure 9a. Below we respond to referee A and B’s questions and comments. We hope we have satisfactorily addressed concerns of the referees so that the paper is now acceptable for publication.

Best Regards,

Weiming An

PROBLEMS WITH MANUSCRIPT:  
  
In reviewing the figures of your paper, we note that the following  
changes would be needed in order for your figures to conform to the  
style of the Physical Review.  Please check all figures for the  
following problems and make appropriate changes in the text of the  
paper itself wherever needed for consistency.  
  
Figure(s) [9]  
Please delete the "titles" at the tops of the figures, and incorporate this information into the caption, if desired.

Answer: We have fixed it.

Figure(s) [1,9a]  
  
Physical Review Accelerators and Beams is an all electronic journal.  
Articles are published electronically using the figure and graphics  
files that authors provide.  Your manuscript has a large file size  
associated with the figures.  Our experience is that this type of  
figure leads to slow downloads and extended printing times, thereby  
making the publication less accessible to readers.  
  
You should reduce the file size by using different graphics programs  
and/or bitmapping of images.  A reasonable target size for the pdf  
file for each separate figure file is 1 MB or less.

We reduced the size of the files. Now all of the pdf files for the figures are now less than 1 MB.

----------------------------------------------------------------------  
Report of Referee A -- ZG10155/Zhao  
----------------------------------------------------------------------  
  
REFEREE A - ZG10155   
  
The authors of the manuscript "Emittance Preservation Through Matching   
the Witness Beam in Plasma Wakefield Acceleration" present analytical   
expressions, obtained by using the WKB solution of a single's particle   
motion, to evaluate the beam emittance in an arbitrary adiabatic   
plasma profile by considering a beam without acceleration but with a   
given energy spread. The results are used to evaluate the conditions   
to minimize the the emittance growth for unmatched beams.   
  
The results of the theory are compared with simulations of 3D QuickPIC   
code. The theory is also applied to the FACET II facility. The authors   
propose experiments based on their results to minimize the transverse   
emittance at the exit of the facility and they also investigate the   
emittance growth in an experiment of the same facility with   
self-ionized plasmas, when the plasma is formed in a lithium gas   
contained by a helium buffer gas.   
  
The paper is well written, and the results are, as far as I know, new   
and interesting, in particular the expression of the emittance   
evolution of a beam having a given energy spread, and the conditions   
of a minimal emittance growth for a fixed beam, even if they are valid   
in absence of acceleration. Therefore, I think it could be published   
in Physical Review Accelerators and Beams if the following points are   
clarified:   
  
Pag. 1, 6 lines before the end of the abstract: "the emittance growth   
can be still be minimized". There's a typo. 

Answer: Thank you for pointing this out. We have corrected it.

Pag. 1, column 1, 4 lines before the end: "where the witness beam is   
located, not only is there a longitudinal" -> I think "is there"   
should be "there is". 

Answer: We think this sentence is grammatically correct.

Pag. 1, column 2, 6 lines before the end: "force is linear   
(proportional to r) , points". There's a space after the round bracket   
that should be removed. 

Answer: This has been corrected.

Pag. 1, column 2, the phrase "This ensures that the beam particles   
will not gain additional slice energy spread when undergoing   
acceleration". While it is clear how this is related to the fact that   
the longitudinal electric field does not depend on r, it is not clear   
why it is also related to a linear transverse force. Maybe the phrase   
should be reformulated (or clarified). 

Answer: We have reformulated this sentence as:

…and does not depend on = ct - z inside the bubble[3]. The fact that the accelerating field does not depend on r ensures that the beam particles will not gain additional slice energy spread when undergoing acceleration and betatron oscillations inside the bubble. Also, the fact that the transverse linear focusing force does not depend on ensures that the beam particles at different longitudinal positions will oscillate at the same betatron frequency, if they have the same energy. If one of these properties is satisfied then the Panofsky Wenzel theorem(\cite{PWT}) guarantees that the other is as well.

Pag. 2, column 1, line 15: the line starts with a comma. 

Answer: This has been corrected.

Pag. 2, column 1, line 28: "Therefore the beam parameters need to be   
changed to match the beam to the plasma". Maybe "optimized" is more   
appropriate than "changed". 

Answer: We have replaced ‘changed’ with ‘optimized’.

Pag. 2, column 1, line 33: "if the witness beam parameters are fixed".   
This phrase should be clarified. What do the authors mean by "fixed"?   
It seems that it is not possible to change the witness beam   
parameters, however, few lines below, they write to change the beam   
focal plane. This is an important point because one relevant result of   
the paper is the condition to inject a beam in order to minimize the   
emittance growth. It is not very clear why it is possible to change   
the focal plane but not to inject a matched beam. 

Answer: We have added a sentence that gives an explanation for how the focal plane is changed,

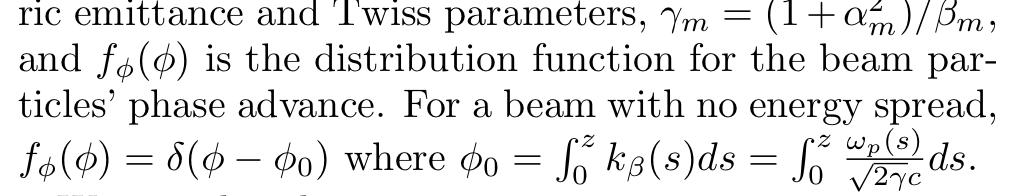
“*We also discuss how to choose the relative focal plane by either moving the plasma or the focal position of the beam to minimize the emittance growth for an unmatched beam with fixed parameters.* ”

Pag.2, column 2, Eq. (2). It could be useful, if possible, also to   
write the condition for the plasma density profile n\_p(z).

Answer: We have added the adiabatic condition in terms of the plasma density as well.

Pag. 3, column 1, 4 lines before Eq. (7) (now it is Eq. (8)), when f\_phi(phi) is defined.   
It is written in Appendix A, but here it should be better explained   
the meaning of this function. Moreover, in the following phrase, the   
average gamma symbol is used, but it is explained after Eq. (9). Also,   
after Eq. (12), there is the explanation of the average phi, which is   
already used here. I think it's better to explain and clarify here all   
these symbols.

Answer: We have modified the sentence in question to the following:



Pag. 3, column 2, before Eq. (9) (now it is Eq. (10)). Instead of "for arbitrary   
f\_phi(phi)" I would write "for arbitrary, but small, energy spread". 

Anwser: We have changed it to “ … for arbitrary fφ(φ) with small energy spread.”

Pag. 3, column 2, 4 lines after Eq. (9) (now it is Eq. (10)).: the derivation of the phase   
advance as a function of gamma for small energy spread, phi(gamma),   
should be added. 

Answer: We have added the derivation in the appendix C.

Pag. 3, column 2, 4 lines before the end: a comment of the   
approximation of neglecting the dependence of the Twiss parameters on   
energy spread should be added. Where it exactly comes from and what   
are the limits of such an approximation. Something is clarified in   
Appendix A, but here a phrase would facilitate a reader.

Answer: We have modified the sentence that begins with, “Note that all of …” to three sentences, “*Note that in order to keep the analysis tractable we have only kept the effects of the energy spread in the betatron phase advance and not on the amplitude of the betatron oscillation in the elements of the transport matrix. The amplitudes are functions of the local values of the Twiss parameters while the phase is an integral in z over 1/. Therefore, only the phase terms can deviate substantially between particles with small energy differences.* ”

Pag. 4, column 1, Fig. 2: is the 5% of energy spread also used for figures (c) and (d)? This should be written. 

Answer: We have changed the caption for Figure 2 accordingly.

Pag. 4, column 1, last line before the expression of A for a uniform   
plasma: "we have alpha\_{mi} = 0 ..." -> "we have alpha\_m = alpha\_{mi}   
= 0, so gamma\_m = gamma\_{mi} = ..."

Answer: We have corrected it.

Pag. 5, column 1, Eq. (17) (now it is Eq. (20)): The authors should add a phrase to explain how these conditions are obtained and under which hypothesis.

Answer: Eq. 18 and the unnumbered equation above it follow from the evolution of a beam propagating in vacuum. We have defined z=s at the focus and z=0 at the entrance and Eq. 18 follows from these choices. To make this clearer we have added a citation and modified the text from, *“..at the focal plane. Therefore, the beam’s initial…” to “…at the focal plane. According to the evolution of Twiss parameters in a drift space [13], the beam’s Twiss parameters at the plasma entrance (z=0) are,”*

Pag. 5, column 1, after Eq. (18) (now it is Eq. (21)): the authors should give examples on how to change the beam's focal plane and comment why it could be   
possible to change the focal plane but not to match the beam. 

Answer: We have modified the introduction to this section to clarify that the beta\* of the beam is assumed to remain fixed but that the plasma profile can change or the location of the focal plane can change. The conclusion that the emittance is always a minimum for the original focal plane therefore means there is no need to actually move the focal plane if the location of the plasma entrance does not change.

Pag. 5, column 2, line 11: "assumptions(adiabatic". There should be a   
space after "assumptions".

Answer: We have fixed it.

Pag. 5, column 2, lines 13, 15, 21 and 22: Figs. 4 should come after   
Figs. 3. 

Answer: We have fixed it.

Pag. 6, column 2, 13 lines before the end: it's not clear if Fig. 6 is   
obtained without acceleration and how important acceleration is on the   
experiment that can be performed at FACET II. Moreover, it's not clear   
if the nominal parameters of FACET II are close or far from the   
conditions they have determined. Is the proposed experiment something   
particular? Is it possible to optimize the nominal FACET II beam, or   
is it already optimized?

Answer: Fig. 6 is indeed obtained with acceleration, which was said ‘This time we turn on the longitudinal push…’. We believe the acceleration is not very important for this case, because the acceleration gradient is constant along for a majority of the witness bunch. This is consistent with the fact that for the optimal case for figure 6 (the black curve), there is almost no emittance growth even though there is acceleration. The beam parameters at FACET-II are not precisely known even at this time. However, the parameters we simulated were chosen to be near the expected ones. For these parameters, we showed the optimal focal point which is likely to change as the parameters are more precisely known.

Pag. 8, Fig. 8: it would be interesting to see what happens of Fig. 8   
(a) if the witness beam's focal plane location is different from 3.39   
cm. How sensible is the emittance growth to this condition with more   
realistic parameters (including acceleration and asymmetry)?

Answer: We have shown that in Fig 6 (a) and (b), and the simulations for Fig 6 have beam acceleration and asymmetric drive beam.

We agree that this would be interesting. We have studied the tolerance of the emittance on the focal plane for round beams including acceleration in preformed plasma. This was shown in figure 6 (a) and 6 (b). For the parameters we are simulating, we do not expect much difference for self-ionized plasma. We leave a detailed study on the optimized focal plane for asymmetric beams for future work.

Pag. 8, column 1, 8 lines before the end: it's true that with an   
initial emittance of 20 um, this remains almost constant, but one   
could argue that this emittance is, in any case, larger than those of   
Fig. 8 (a). What is then the advantage? The authors should add a   
comment on this.

One goal of the experiment is to show that the emittance of a matched beam can be preserved. Because of the buffer gas and self-ionization, this can only be demonstrated using emittance of 20um. Once this has been demonstrated, a second experiment using laser ionization can be conducted to show emittance preservation with much smaller initial emittance. This comment has been added to the text.

Pag. 9, column 1, line 30: "We find that this can potentially lead to   
the witness beam’s emittance can growing by". The phrase seems to   
contain a typo.

Answer: We have deleted the second ‘can’ in the sentence.  
  
Pag. 9, column 2, line 3: "above integral we assume the". I would add:   
"above integral with an energy spread in the beam, we assume the".

Answer: We have added ‘with an energy spread in the beam’ in the sentence.

----------------------------------------------------------------------  
Report of Referee B -- ZG10155/Zhao  
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REFEREE B - ZG10155   
  
This paper presents a theoretical analysis concerning the emittance   
evolution of a beam in a plasma wakefield accelerator (PWFA) when   
density ramps at the entrance and exit of the plasma target are taken   
into account. The authors derive an analytical expression (assuming   
there is no acceleration and the plasma profile is adiabatic) for the   
beam emittance growth when the beam has an energy spread and it is not initially matched. Furthermore, given a beam with fixed initial   
parameters and a fixed plasma density ramp, the authors determine to   
optimal position of the beam's focal plane in vacuum such that the   
emittance growth is minimized when the beam enters the plasma.   
Analytical results are validated by means of numerical modeling   
performed with the code QuickPIC. Finally, the authors consider   
emittance degradation from beam-induced ionization of residual gas   
(Helium buffer) when the beam enters the plasma. The study is of   
interest for FACET II.   
  
My first impression was that this paper is a mild extension of the   
works presented in Refs. 4-7. However, the determination of the   
optimal position of the beam's focal plane in vacuum discussed in Sec.   
III is of some interest and, to my knowledge, it was not treated in   
previous literature. Hence, this paper could be considered for   
publication in PRAB. However, before publication can be granted, the   
comments and criticisms discussed below should be addressed and the manuscript amended accordingly. 

We note that we have also expanded Sec. III to include a discussion on how the emittance will vary due to shot to shot variations of the plasma conditions for a fixed focal position.

In Sec. III the authors say that the profile used in Fig 4 was   
obtained by cutting the “non-adiabatic” tail at the entrance and exit.   
How exactly was this done? The adiabaticity condition Eq. (2) [or Eq.   
(5)] does not identify a precise point (the condition is expressed by   
“<< 1”, and not a “< 1”). How sensitive are the results to the exact   
location of the cut? Making the cut at, let's say, +/-1 cm from the   
chosen position does it change the results significantly? Was the non   
adiabatic part of the profile considered in the QuickPIC simulation   
(Fig. 3). Please clarify.

Answer: We agree that this requires clarification. The “non-adiabatic” region was not rigorously defined. We found a region that was 70cm long such that at the adiabatic condition was satisfied at both the entrance and exit. By the adiabatic condition we required that || << 1. We chose the plasma to be between 5cm and 75 cm in this case, where ||=0.24 at 5cm and 0.56 at 75cm respectively. We clarified this in the text. We do not expect the results to change if the entrance and exit are moved, but have not done a detailed study of how the results agree with the adiabatic theory if || is closer to unity near the entrance or exit. We leave this for future work. The non-adiabatic profile (full) is shown in figure 3a and it was used to generate the results in fig. 6. The adiabatic profile (cut) is shown in figure 3b and it was used to generate the results in fig. 4.

Equation (20) (now it is Equation (24))is not justified nor derived anywhere in the text. 

Answer: We have added the derivation in the appendix.

Why are the Twiss parameters in Eq. (19) (now it is Equation (23)) [Sec. IV] different from the ones given at the end of Sec. III (the density profile and witness   
beam parameters seem the same in both sections)? In general more   
clarity on the way parameters are chosen would be greatly appreciated. 

Answer: The difference is that the values in Eq. 19 are the matched values at the entrance of the full profile while the values at the end of Sec. III are the values at the entrance of the truncated (adiabatic) profile. We have clarified this in the text.

The issue discussed in Sec. V was already discussed in Bruhwiler et   
al., PoP 10, 2022 (2003). It is surprising that the authors did not   
acknowledge this work. Increasing the initial beam emittance (from 3   
mm mrad to 20 mm mrad) to suppress the emittance growth is not really an elegant solution since now, even though the emittance growth is suppressed, the quality of the bunch is intrinsically bad from the start. 

Answer: We are familiar with the work of Bruhwiler et al., but to our knowledge the issue discussed in Sec. V, i.e., that the density profile will change due to specific parameters chosen at FACET II leading to emittance growth, were not discussed. The parameters used in experiments in PWFA have changed substantially since this earlier work. However, we have now included a reference to Bruhwiler et al. when plasma formation due to self-ionization is first mentioned.

The list of references in this paper is very FACET/UCLA-centric, this   
degree of self-referentiality is a bit disconcerting. At a very   
minimum, the authors should add “Bruhwiler et al., PoP 10, 2022   
(2003)” to the list of references. The problem of emittance   
preservation/degradation when a beam is injected or extracted from a   
plasma stage has been analyzed by several groups and this effort, I   
think, should be acknowledged. A list of possible relevant references   
should include Mehrling et al., PRSTAB 15, 111303 (2012); I. Dornmair   
et al., PRSTAB 18, 041302 (2015); P. Antici et al., J. Appl. Phys.   
112, 044902 (2012); M. Migliorati et al., Phys. Rev. ST Accel. Beams   
16, 011302 (2013). In addition, the authors should/could acknowledge   
related work in the contest of conventional accelerators (e.g., check   
the reference list in Ref. 7). 

Answer: We have included these references.

The language and text should be revised removing typos (e.g., there   
are two typos in the abstract) and improving the form of the English   
language (e.g., the title does not read particularly well) is strongly   
encouraged.

Answer: We have fixed those typos.